

The masses of the mesons and baryons. Part IV. Integer multiple rule extension

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It is shown that the empirical rule that the masses of the stable mesons and baryons of the γ -branch are integer multiples of the mass of the π^0 meson with a maximal deviation of 3.3% holds also for the meson and baryon resonances, regardless whether the spin of the particles is 0 or $\frac{1}{2}$. It is also shown that the masses of the particles with weak decay are integer multiples of the mass of the π^\pm mesons times a common factor 0.853.

1 Introduction

In a previous article [1] we have shown that the masses of the so-called stable mesons and baryons of the γ -branch are integer multiples of the π^0 meson, with a maximal deviation of 3.3% and an average deviation of 0.73%. The γ -branch of the stable mesons and baryons consists of the particles whose decay is electromagnetic, the characteristic example of the γ -branch is the π^0 meson which decays via $\pi^0 \rightarrow \gamma\gamma$ (98.8%). The other branch of the mesons and baryons is the neutrino branch. The weak decay of the particles of the ν -branch is accompanied by the emission of neutrinos, the characteristic examples are the π^\pm mesons which decay via e.g. $\pi^+ \rightarrow \mu^+ + \nu_\mu$ (99.987%). The discussion in [1] was limited to the so-called stable particles. We have explained the integer multiple rule of the stable mesons and baryons of the γ -branch with the standing wave model proposed in [2,3]. We will now show that the integer multiple rule does not only apply to the stable particles, but also to the meson and baryons resonances with isospin $I \leq 1$ and spin $J \leq \frac{1}{2}$.

2 The spectrum of the γ -branch particles

From a theoretical point of view it is clear that the particles with the most simple structure are those with $I, J = 0, 0$, but also with strangeness $S = 0$ and charm $C = 0$. All $I, J = 0, 0$ mesons of the γ -branch, whether they are stable or resonances, are listed in Table 1, which is based on the Particle Physics Summary [4]. The π^0 meson does not appear in Table 1 because its isospin $I = 1$. Also omitted are two f_0 resonances with $I, J = 0, 0$ whose mass is uncertain by more than $\pm 3\%$. Included in this Table are two charmed particles, the η_c

Table 1: The γ -branch mesons with I, J = 0, 0

particle	m/m(π^0)	N		particle	m/m(π^0)	N
η	4.0559	4		$\eta(1440)$	10.48	10
η'	7.0958	7		$f_0(1500)$	11.135	11
f_0	7.261	7		η_c	22.076	22
$\eta(1295)$	9.594	10		χ_{c0}	25.301	25

and χ_{c0} mesons; bottom particles are not considered. A least square plot of the masses of the γ -branch mesons with I, J = 0, 0 as a function of the integer N is shown in Fig. 1. The integer N is the integer number nearest to the actual ratio m/m(π^0). The points in Fig. 1 refer to resonances, but for the point at N = 4 which represents the η meson.

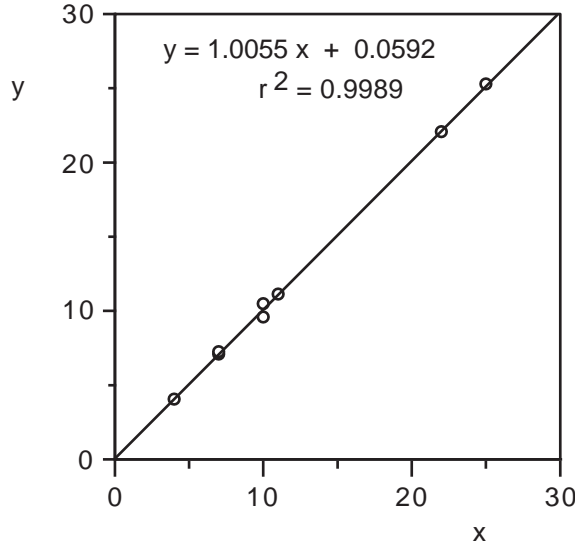


Fig.1: The mass of the γ -branch mesons in units of m(π^0) as a function of the integer N. $y = m/m(\pi^0)$; x is integer N.

As Fig. 1 shows the I, J = 0, 0 γ -branch mesons follow the integer multiple rule very well. According to this figure the masses are determined by the formula

$$m(N)/m(\pi^0) = 1.0055 N + 0.05923, \quad (1)$$

with the almost perfect correlation coefficient 0.999. The difference between the line described by (1) and the ideal line for integer multiples, which goes through the origin and has slope 1.0000, may originate from an improper choice of the reference particle. The experimental value of m(π^0) is known to seven

decimals, but the π^0 meson is a particle with $I = 1$, and therefore not necessarily the proper reference for a line describing the $I, J = 0, 0$ mesons. We do not know how the value of the isospin affects the mass of a particle. From Eq. (1) follows that $m(1) = 1.0647 m(\pi^0)$. Presumably this is the mass of a neutral meson with $I, J = 0, 0$. It is, by all means, possible that $m(1)$ with $I, J = 0, 0$ has a mass 6.5% larger than $m(\pi^0)$ with $I, J = 1, 0$, or whose mass is 3% larger than $m(\pi^\pm)$. The majority of the charged stable particles have a mass smaller than the mass of the corresponding neutral particles. From the masses of the stable particles it is also apparent that the differences between charged and neutral particles is particularly large (in percent) in the case of the π mesons. According to Eq. (1) the contribution of the intercept $y(0)$ to the masses of the particles decreases with increased N in agreement with the empirical fact that the higher N the smaller are, in general, the deviations of the particle masses from strictly integer multiples of $m(\pi^0)$.

Next we look at the masses of the stable baryons and of the baryon resonances as a function of N . The masses of the γ -branch baryons with $I \leq 1, J = \frac{1}{2}$ are listed in Table 2.

Table 2: The γ -branch baryons with $I \leq 1, J = \frac{1}{2}$

particle	$m/m(\pi^0)$	I, J	N	particle	$m/m(\pi^0)$	I, J	N
Λ	8.26577	$0, \frac{1}{2}$	8	Ξ^0	9.7417	$\frac{1}{2}, \frac{1}{2}$	10
$\Lambda(1405)$	10.42	$0, \frac{1}{2}$	10	Λ_c^+	16.928	$0, \frac{1}{2}$	17
$\Lambda(1670)$	12.37	$0, \frac{1}{2}$	12	$\Lambda_c(2593)$	19.215	$0, \frac{1}{2}$	19
$\Lambda(1800)$	13.33	$0, \frac{1}{2}$	13	Σ_c^0	18.167	$1, \frac{1}{2}$	18
Σ^0	8.835	$1, \frac{1}{2}$	9	Ξ_c^0	18.302	$\frac{1}{2}, \frac{1}{2}$	18
$\Sigma(1660)$	12.298	$1, \frac{1}{2}$	12	Ω_c^0	20.03	$0, \frac{1}{2}$	20
$\Sigma(1750)$	12.965	$1, \frac{1}{2}$	13				

The Ω^- particle has been omitted from the list because it has spin $\frac{3}{2}$. Including however Ω^- in the list does not alter the following results significantly. Also omitted in Table 2 is the $\Lambda(1810)$ resonance which differs from $\Lambda(1800)$ only in parity, and two Ξ resonances whose spin is uncertain. A least square analysis of the masses in Table 2 yields the formula

$$m(N)/m(\pi^0) = 1.0013 N + 0.1259, \quad (2)$$

with the very good correlation coefficient 0.997. The baryons and baryon resonances follow the integer multiple rule in a good approximation. The comparatively large intercept in Eq. (2) is of less concern than in the case of the mesons because the first mass affected by $y(0)$ is that of the Λ baryon with $N = 8$, which means that $y(0)$ contributes only 1.57% to $m(\Lambda)$. To assess the significance of the difference between $m(\Lambda)$ according to Eq. (2) and the measured $m(\Lambda)$ we must keep in mind that the Λ baryon has spin $\frac{1}{2}$ and strangeness $S = -1$, whereas

spin and strangeness of the reference particle π^0 are both zero. If we combine all mesons and baryons of the γ -branch with $I \leq 1, J \leq \frac{1}{2}$ of Tables 1,2 we arrive at Fig. 2 which shows that these 22 particles, eight $I, J = 0, 0$ mesons, thirteen $J = \frac{1}{2}$ baryons and the π^0 meson with $I, J = 1, 0$ follow the integer multiple rule in a good approximation with the nearly perfect correlation coefficient 0.999. It is

$$m(N)/m(\pi^0) = 1.0056 N + 0.061026, \quad (3)$$

which differs only marginally from Eq. (1). This is so although the line on Fig. 2 describes stable and unstable particles with different spin, $J = 0$ for the mesons and $J = \frac{1}{2}$ for the baryons. Spin $\frac{1}{2}$ should make a contribution to the energy of the particles but, as Fig. 2 shows and as we pointed out in [1], spin $\frac{1}{2}$ does not alter the integer multiple rule. So do different values of the isospin or different values of the strangeness, in particular $S \neq 0$ for Λ , Σ , Ξ , and so do different values of $C \neq 0$ of the charmed baryons and the η_c and the χ_{c0} mesons as well. It is astounding that the integer multiple rule holds in spite of the differences in the four parameters involved and regardless of whether the particles are stable or resonances.

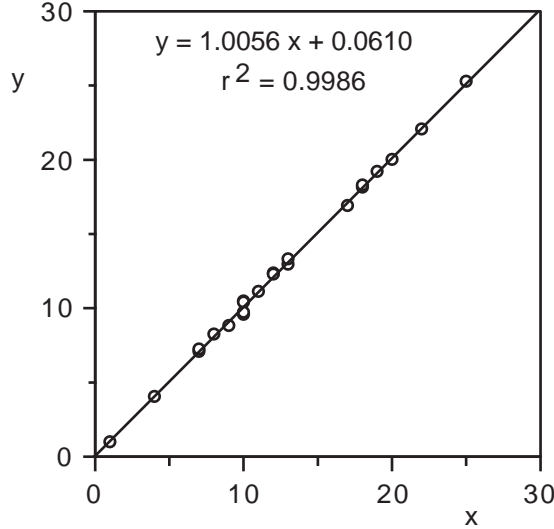


Fig. 2: The mass of all mesons and baryons with $I \leq 1, J \leq \frac{1}{2}$ in units of $m(\pi^0)$ as a function of the integer N . $y = m/m(\pi^0)$; x is integer N .

3 The spectrum of the ν -branch particles

So far we have been concerned with the γ -branch of the particles. The neutrino branch is clearly distinguished from the γ -branch by the neutrinos which are

emitted when the particles of the ν -branch decay. The particles of the ν -branch, according to the Particle Physics Summary, are listed in Table 3 together with the neutron resonances.

Table 3: The particles of the ν -branch

particle	$m/m(\pi^\pm)$	I, J	N	particle	$m/m(\pi^\pm)$	I, J	N
π^\pm	1.0000	1, 0	1	N(1535)	10.998	$\frac{1}{2}, \frac{1}{2}$	13
K^\pm	3.53713	$\frac{1}{2}, 0$	4	N(1650)	11.822	$\frac{1}{2}, \frac{1}{2}$	14
n	6.73186	$\frac{1}{2}, \frac{1}{2}$	8	D^\pm	13.393	$\frac{1}{2}, 0$	16
N(1440)	10.317	$\frac{1}{2}, \frac{1}{2}$	12	D_S^\pm	14.104	0, 0	16

The reference particles for the ν -branch appear to be the π^\pm mesons because they are typical for the weak ν -branch decays. As we have pointed out in [1] the ratios of the masses of the ν -branch particles to the mass of the π^\pm mesons are integer numbers N times a common factor 0.861 ± 0.022 . The ratios cannot be straight integer multiples of $m(\pi^\pm)$ because $m(K^\pm)/m(\pi^\pm) = 0.8843 \cdot 4 = 3.537$.

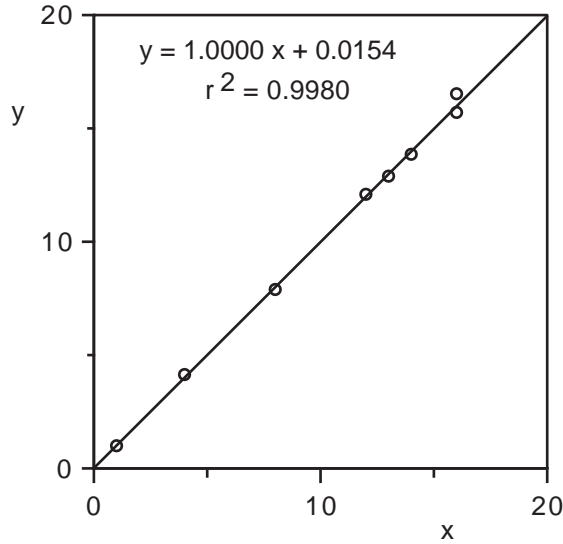


Fig. 3: The mass of the neutrino branch particles in units of $m(\pi^\pm)$ as a function of the integer N. $y = m/0.853m(\pi^\pm)$; x is integer N.

A least square analysis of the masses in Table 3 with $m(N)/0.861m(\pi^\pm)$ versus N does, however, not have a line with slope 1. But a plot of $m(N)/0.853m(\pi^\pm)$ versus N ($N > 1$) has slope 1.0000 and a small intercept $y(0)$ as shown on Fig. 3. The masses of the neutrino branch are therefore given by

$$m(N)/0.853 m(\pi^\pm) = 1.0000 N + 0.01537, \quad N > 1, \quad (4)$$

with the correlation coefficient $r^2=0.998$. If we drop the neutron resonances in Table 3 we find

$$m(N)/0.853 m(\pi^\pm) = 1.0055 N + 0.00575, \quad N > 1.$$

The slope 1.0055 in this formula can, of course, be reduced to 1.0000 by proper choice of the factor before $m(\pi^\pm)$, i.e. by 0.8574. The common factor 0.853 in Eq. (4) differs by only 1% from the empirical value 0.861 given in [1], well within the uncertainty ± 0.022 of 0.861 given there. The mass $m(1)$ of Eq. (4) is, by definition, the same as $m(\pi^\pm)$, the mass $m(4)$ is $0.969 \cdot m(K^\pm)$ and $m(8)$ is $0.984 \cdot m(n)$. The 3.1% difference of $m(4)$ from $m(K^\pm)$ may be the result of the strangeness $S = \pm 1$ and of $I = \frac{1}{2}$ of the K mesons, whereas the reference particle π^\pm has $S = 0$ and $I = 1$, in other words the π^\pm mesons are not the perfect reference for the K^\pm mesons.

Surprisingly the π^0 meson provides also a good reference for the ν -branch particles. A least square analysis of the eight ν -branch particles of Table 3 gives the line

$$m(N)/m(\pi^0) = 1.0000 N - 2.284 \cdot 10^{-4}, \quad (5)$$

with the still very good correlation coefficient $r^2=0.996$. Equation (5) makes it appear that the particles of the ν -branch are integer multiples of $m(\pi^0)$. However, the mass of $m(4)$ according to Eq. (5) is $1.0936 \cdot m(K^\pm)$, which is a much worse fit to $m(K^\pm)$ than $m(4)$ according to Eq. (4), whose $m(4) = 0.968 \cdot m(K^\pm)$. On the other hand $m(7)$ according to Eq. (5) is only 0.57% larger than $m(n)$. But neither the K^\pm mesons nor the neutron decay electromagnetically as the reference π^0 meson does. Rather both particles decay with the emission of neutrinos, K^\pm in 71.5% of the cases and the neutron in 100% of the cases, and we believe that the decays tell what the particles are made of. Actually the place $N=4$ in Eq. (5) or the place for a particle with four times the mass of the π^0 meson is filled with the η meson with the measured $m(\eta) = 1.014 \cdot 4 m(\pi^0)$. The η meson however is, as its decay shows, part of the γ -branch of the particles. Therefore Eq. (4) describes correctly the neutrino branch and not Eq. (5). The factor 0.853 in front of $m(\pi^\pm)$ in Eq. (4) will be explained in a forthcoming paper as a consequence of the oscillations of a neutrino lattice.

4 Particles with spin 1

From the foregoing it might appear that the masses of the particles are always integer multiples of a particular reference particle, or integer multiples of the mass of a reference particle times a constant factor. However the meson resonances with spin 1 tell that this is not necessarily so. The meson resonances with spin 1 and $S, C = 0, 0$ according to [4] are listed in Table 4. This table contains the pair b_1 and a_1 as well as the pair $f_1(1420)$ and $\omega(1420)$ whose particles differ from another in their parities, but in each pair the masses of the particles

Table 4: Meson resonances with spin 1

particle	$m/m(\pi^0)$	I, J	N	particle	$m/m(\pi^0)$	I, J	N
$\rho(770)$	5.694	1, 1	6	$f_1(1420)$	10.571	0, 1	10
$\omega^0(782)$	5.7932	0, 1	6	$\omega(1420)$	10.513	0, 1	10
$\phi^0(1020)$	7.55253	0, 1	8	$\rho(1450)$	10.854	1, 1	11
$h_1(1170)$	8.668	0, 1	9	$f_1(1510)$	11.202	0, 1	11
$b_1(1235)$	9.1201	1, 1	9	$\omega(1600)$	12.217	0, 1	12
$a_1(1230)$	9.1127	1, 1	9	$\phi(1680)$	12.447	0, 1	12
$f_1(1285)$	9.4994	0, 1	9	$\rho(1700)$	12.595	1, 1	13

are nearly the same. Considering in the least square analysis only one mass of each pair increases the slope and the intercept in Eq. (6) substantially. A least square analysis of the 14 particles in Table 4 yields a straight line given by

$$m(N)/m(\pi^0) = 1.0257 N - 0.3345, \quad r^2=0.977. \quad (6)$$

The intercept $y(0)$ in (6) is much larger than the intercepts in Figs. 1,2 and the difference of the correlation coefficient from a perfect 1.0000 is by an order of magnitude larger than in Figs. 1,2. It seems to be questionable that the masses of the meson resonances with spin 1 can be described satisfactorily by an integer multiple rule.

5 Conclusions

We have found that the masses of the stable mesons and of the meson resonances of the γ -branch are a linear function of the variable N which denotes integer multiples of the mass of the π^0 meson. The correlation between $m(N)$ and N has a nearly perfect correlation coefficient $r^2=0.999$. The same applies when the mesons and meson resonances with $I \leq 1, J=0$ are combined with the baryons and baryon resonances with $I \leq 1, J=\frac{1}{2}$. Spin $\frac{1}{2}$ does not seem to affect the integer multiple rule, neither does strangeness nor charm. We have, furthermore, confirmed our previous finding that the masses of the particles with weak decay, i.e. of the ν -branch, are integer multiples of the π^\pm mesons times a common factor. This rule holds with the near perfect correlation coefficient $r^2 = 0.998$.

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